Outage-Capacity Tradeoff for Smart Grid with Renewables

[Extended Abstract]

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ABSTRACT
Future power grid systems are envisioned to be integrated with many distributed renewable energy sources (DRES). Energy storage is the key technology to enable reliable and cost-effective renewable energy. Given the fact that large-scale energy storage device is usually costly to install and operate, we are naturally led to the following question: How much storage is needed to guarantee the stability of a power grid network with DRESs? This paper represents a first step in systematically exploring the tradeoff between the capacity of energy storage devices and the outage probability, i.e., the probability of the occurrence of imbalance between the supply and demand. We first propose a secure scheduling and dispatch (SSD) algorithm that is capable of maintaining the grid stability in the presence of volatility in the power generation. We then derive a closed-form bound to quantify the tradeoff between the storage capacity and the outage probability. Under mild assumptions, we reveal that the outage probability decreases exponentially with respect to the square of the storage capacity. This finding implies that energy storage is an effective and economically viable solution to maintain the stability of a smart grid network even in the presence of many volatile and intermittent renewable energy sources. The impact of correlation in energy generation on the stability of a smart grid network is also investigated.

Keywords
Renewable energy sources, wind and solar power, energy storage, scheduling and dispatch, demand response, outage control.

1. INTRODUCTION
The balancing of electricity supply and demand is crucial for the reliable operation of a power grid network [1, 2]. The balancing task becomes increasingly important as it is anticipated that a large number of small-scale DRESs will be integrated into the future power grid system. These distributed energy sources could not only be used to serve the local customers that are directly connected to them but also supply energy to the utility companies during the peak time. In addition, as DRESs are distributed in nature, they could get disconnected from the main power grid in case an outage occurs, which could considerably enhance the stability and reliability of the local power grid. Renewable energy sources such as the solar, wind, bio-fuel, and geothermal energy are popular DRESs. While such renewable energy sources have the potential of significantly saving transmission energy and reducing CO2 emissions, the amount of energy provided by wind and solar power are highly unpredictable and time-varying, which may lead to either outage of the power grid or poor power quality. It has been reported that outage and power quality issues on average costs billions of dollars in US every year. The unpredictable and time-varying nature of the renewable energy could severely limit the successful operation of the smart grid system.

In this paper, we study the scheduling and dispatch approaches for a power grid system with DRESs and energy storage devices in the presence of supply uncertainties. In particular, a statistical model is presented to capture the uncertainties in the power supply. Two sources of uncertainties are of particular interest. The first source of uncertainty comes from perturbation of the amount of energy generated by the DRESs from the generation schedule due to factors like the change of the wind speed and the sunlight intensity. The second one is the disruptive uncertainty on the power grid system, such as failure of a DRES or the disconnection of a local grid from the main grid. We then formulate the scheduling and dispatch problem as a non-convex optimization problem and develop optimal solution to this problem via converting it into a convex optimization problem. The tradeoff between the energy storage capacity and the grid stability is quantified by deriving a closed-form bound to the outage probability of a power grid network with given energy storage capacity.

The remainder of this paper is organized as follows. In Section 2, We propose a statistical model to characterize different types of uncertainties in the power generation. Based on this model, we formulate the secure scheduling and dispatch problem as a non-convex optimization problem. In Section 3, the tradeoff between the outage probability and the energy storage capacity is studied. Numerical results are presented in Section 4 and we conclude in Section 5.
2. PROBLEM FORMULATION

We consider a power grid consisting of $M$ DRESs. Analogously to [3], an slotted time model is assumed for energy balancing. Let $d(t)$ denote the aggregated energy demand in the $i$th time slot. $x_i(t)$ is the amount of energy the $i$th DRES is scheduled to generate. $x_i^{\max}(t)$ is the maximum amount of energy the $i$th DRES can generate during the $i$th time slot. $c_i(x_i, t)$ denotes the cost function of the $i$th DRES.

The goal of the utility company is to minimize its total cost while satisfying the demand, as shown below,

$$\text{minimize} \quad \sum_i c_i(x_i, t) \quad \text{(1)}$$

subject to

$$\sum_i x_i(t) = d(t), \quad x_i(t) \leq x_i^{\max}(t),$$

variables $x_i(t) \geq 0.$

For notational simplicity, we omit the index $t$ hereafter. Recall that $x_i$ is the generation schedule of $i$th DRES, namely, the amount of energy the $i$th DRES is scheduled or predicted to provide. $\bar{x}_i$ is the actual load of the $i$th DRES. The difference between the generation schedule and the real load can be characterized by: $\bar{x}_i = (1 + g_i)x_i$, where $\{g_i : g_i \in [g_i^l, g_i^u] \geq 0\}$ is a random variable that quantifies the deviation of the generation schedule of the $i$th DRES from the real load, where $g_i^l$ and $g_i^u$ denote respectively the lower and upper bound to $g_i$. In this paper, we focus on two types of uncertainties in the energy generation of DRESs, namely the generator forced outage, and the supply uncertainty in DRES power generation.

**Generator forced outage:** We can set $g_i^l$ as $-1$ and $g_i^u$ as $0$ to account for the effect of the generator forced outage, e.g., the disconnection of a DRES such as a wind turbine and solar panel from the grid due to equipment failure. In doing so, $g_i$ is modeled as a Bernoulli random variable with two realizations, namely $-1$ and $0$.

**Supply uncertainty in DRES power generation:** Recall that $g_i$ quantifies the difference between the generation schedule and the real load of a DRES. For supply uncertainty, we assume that the $g_i$ follows a continuous and symmetrical distribution with zero mean or median. Such an assumption complies with the experimental findings obtained in reality and also is general enough to capture a wide range of probability distributions. For example, for wind power generation, $g_i$ can be viewed as the forecast error under the day-ahead point forecast model [4]. Recent work has indicated the forecast error follows continuous and symmetrical distributions, e.g., Gaussian or Cauchy distribution. And the forecast error is one fraction of the wind power installed capacity, which means $g_i^l \geq -1$ and $g_i^u \leq 1$ [4]. Also, the probability distribution information is usually available one day ahead by the energy management system of the utility company.

Before we proceed with further discussion, we first give the definition of the global outage probability as follows,

**Definition 1. Global Outage Probability:** The global outage probability (GOP) of a smart grid system is defined as the probability that the power surplus/deficit of the smart grid system exceeds given thresholds

$$P^G = P^G_c + P^G_u = P\left(\sum_i g_i x_i \leq l_o\right) + P\left(\sum_i g_i x_i \geq u_o\right) \quad \text{(2)}$$

Note that $l_o$ and $u_o$ determine the maximum volatility a given power grid system can sustain. The role of $P^G$ is to mathematically quantify the chance of the occurrence of the instability of a power grid system.

$$-d_{\bar{a}}(t) \leq \sum_i \bar{x}_i(t) - d(t) \leq d_{\bar{a}}(t). \quad \text{(3)}$$

For brevity, the time index $t$ is omitted in the following discussion. We assume that the parameters $\{g_{i}\}$ belong to a bounded set termed as confidence region, and we seek to obtain a solution that remains feasible for all realizations within this confidence region, as given in the sequel,

$$\text{minimize} \quad \sum_i c_i(x_i) \quad \text{(4)}$$

subject to

$$\sum_i x_i = d, \quad x_i \leq x_i^{\max},$$

$$p_u(x, \alpha) \leq d_{\bar{a}}, \quad p_l(x, \alpha) \geq -d_{\bar{a}},$$

variables $x_i \geq 0.$

where $x \in [x_1, \ldots, x_i, \ldots]^{T}$, $p_u(x, \alpha)$ and $p_l(x, \alpha)$ are the protection functions given by

$$p_u(x, \alpha) = \max_{S \subseteq \mathcal{I}, |S| = \alpha, g_i \in [g_i^l, g_i^u]} \left(\sum_{i \in S} g_i x_i\right), \quad \text{(5)}$$

$$p_l(x, \alpha) = \min_{S \subseteq \mathcal{I}, |S| = \alpha, g_i \in [g_i^l, g_i^u]} \left(\sum_{i \in S} g_i x_i\right). \quad \text{(6)}$$

$\alpha$ is termed as degree of protection (DoP). The role of the protection function is to guarantee that the resulting solution remains feasible deterministically as long as at most $\alpha$ DRESs are subject to generation uncertainty, e.g., generator forced outage. In addition, even more than $\alpha$ DRESs are subject to generation uncertainty, the solution obtained from (4) remains feasible probabilistically. Also, the outage probability, i.e., the probability that the solution violates the constraints in (4), is a function of $\alpha$. Efficient optimal distributed algorithms have been developed in [5]. Due to page limit, the details of the algorithms are not presented in this paper.

3. OUTAGE-CAPACITY TRADEOFF

We assume an outage occurs if the energy deficit/surplus could not be compensated by the energy storage devices.
Therefore, it is crucial to identify the minimum capacity of the energy storage device that could stabilize the power grid network. Two important analytical results are presented as follows, detailed proofs can be found in [5].

**Definition 2.** For a given \( \alpha \), let \( U(\alpha) \) be the set of \( \{d_{bs}, d_{ul}\} \) such that there is at least one feasible solution to (4). The minimum (energy) storage capacity (MSC) \( d_{bs}(\alpha) \) is defined as follows, i.e.,

\[
d_{bs}(\alpha) = \min d_{bs}, \quad [d_{bs}, d_{ul}] \in U(\alpha).
\]

(7)

Likewise, the minimum operating reserve (MOR) is defined as

\[
d_{ul}(\alpha) = \min d_{ul}, \quad [d_{bs}, d_{ul}] \in U(\alpha).
\]

(8)

**Theorem 1.** MSC and MOR are upper bounded respectively by the product of the DoP and an scalar. In other words, there exists two scalars \( \phi_\alpha \) and \( \phi_\gamma \) such that \( d_{bs}(\alpha) \leq \phi_\alpha \cdot \alpha \) and \( d_{ul}(\alpha) \leq \phi_\gamma \cdot \alpha \).

**Theorem 2.** Assume that \( g_i, 1 \leq i \leq M \), are independent with each other. \( \bar{x} = [\bar{x}_1, ..., \bar{x}_M] \) is the optimal solution to (4) when \( d_{bs} = d_{ul} = d_G = 0 \). Let \( \psi_\alpha = P_{g}(\bar{x}, 1) \), and \( \psi_\gamma = -P_l(\bar{x}, 1) \). We further assume that \( \psi_{\gamma, max} = \max_i \{g_i \cdot x_i^{max}\} \) and \( \psi_{\gamma, min} = -\min_i \{g_i \cdot x_i^{max}\} \). Then the following inequalities hold,

1. For generator forced outage, \( P^{G} = \frac{1}{2} \exp \left[ -\frac{\psi_{\alpha, min} - \psi_{\gamma, min} - 2\psi_{\gamma, max}}{2} \right] \)
2. For supply uncertainty in DRES generation, assuming the pdf of \( g_i \) is symmetric, we have that

\[
P^{G} \leq \exp \left( -\frac{d_{bs}^2}{\eta_\alpha(M)M} \right) + \exp \left( -\frac{d_{ul}^2}{\eta_\gamma(M)M} \right)
\]

\[
\leq \exp \left( -\frac{d_{bs}^2}{\psi_{\gamma, max}^2 M} \right) + \exp \left( -\frac{d_{ul}^2}{\psi_{\gamma, max}^2 M} \right)
\]

where

\[
\eta_\alpha(M) = \frac{\rho_g(\bar{x}, M)}{\rho_g(\bar{x}, M)} \quad \text{and} \quad \eta_\gamma(M) = \frac{\rho_l(\bar{x}, M)}{\rho_l(\bar{x}, M)}.
\]

The above theorem reveals the inherent tradeoffs between the energy storage capacity, the energy reserve, and the outage probability.

### 3.1 Correlated Source of Uncertainty

The proposed approach can be extended to deal with the corrected sources of uncertainties as well. In particular, our analysis shows that the correlation has significant impact on MSC and MOR i.e., significant amount of additional energy storage/operating reserve is required to maintain a target outage probability [5].

### 4. NUMERICAL RESULTS

The tradeoff between the outage probability \( P^{G} \) and Minimum storage capacity (MSC) of a smart grid network is presented in Figure 1. For clarity, the MSC is normalized by the total demand. It is shown that the outage probability can be driven down to smaller than 1e−8 if the relative energy storage capacity is only 0.25%. In addition, as evident from the figure, outage bound 1 and outage bound 2 are almost identical in this case.

![Figure 1: Tradeoff between the outage and storage capacity when \( x_{i}^{max} = 2.5 \times 10^{-4} \).](image)

5. CONCLUDING REMARKS

Energy storage is the key enabling technology to ensure the reliable operation of a power grid network with renewable energy sources. In this paper, we have developed scheduling and dispatch schemes in the presence of volatile DRESs and mathematical quantified the tradeoff between the outage probability and energy storage capacity. The engineering implication of this finding is that in spite of the unpredictable and volatile nature of DRESs, fast-response energy storage device is a viable means to stabilize the power grid.

### 6. REFERENCES


