Distributed Lagrangian Method for Tie-Line Scheduling in Power Grids under Uncertainty

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ABSTRACT

System operators (SOs) manage the grid and its assets in different parts (areas) of an interconnected power network. One would ideally seek to co-optimize the grid assets across multiple areas by solving a centralized optimization problem. Gathering the dispatch cost structures and the network constraints from all areas for a centralized solution remains difficult due to technical, historical, and sometimes legal barriers. Motivated by the need for a distributed solution architecture for multi-area power systems, we propose a distributed Lagrangian algorithm in this paper. We establish convergence rates for our algorithm that solves the deterministic tie-line scheduling problem as well as its robust variant (with policy space approximations). Our algorithm does not need any form of central coordination. We illustrate its efficacy on IEEE test systems.

1 INTRODUCTION

Motivated by the question of optimal tie-line scheduling in electric power systems, we study a distributed algorithm to solve an optimization problem of the form

$$\begin{aligned} \mathsf{P}: \quad \text{minimize} \quad & \sum_{i=1}^{n} f_i(x_i, y_i), \\ \text{subject to} \quad & (x_i, y_i) \in \mathcal{S}_i, \qquad (1a) \\ & \sum_{i=1}^{n} \mathbf{A}_i y_i \leq b, \qquad (1b) \end{aligned}$$

over the variables $x_i \in \mathbf{R}^{N_i^x}$, $y_i \in \mathbf{R}^{N_i^y}$ for each $i = 1, \ldots, n$, where $\mathbf{R}$ denotes the set of real numbers. Each node i is only privy to $f_i, \mathbf{A}_i, b$, and $\mathcal{S}_i$, where

$$f_i \in \mathbf{R}^{N_i^x + N_i^y} \to \mathbf{R}, \ \mathbf{A}_i \in \mathbf{R}^{N \times N_i^y}, \ b \in \mathbf{R}^N, \ \mathcal{S}_i \subset \mathbf{R}^{N_i^x + N_i^y}.$$

Here, each node is an *agent* with computational capabilities and can only communicate with other nodes that are connected to it in an undirected graph $\mathfrak{G}$. We make the following assumptions.

Assumption 1. *(i) $\mathfrak{G}$ is connected. (ii) f_i is jointly convex in its arguments. (iii) $\mathcal{S}_i$ is compact and convex. (iv) Slater's condition holds for* P. *(v) Agent i can optimize f_i over $\mathcal{S}_i$.*

A Lagrangian method [3] has been widely used to obtain a decentralized framework to solve problem P. However, this method requires a central coordinator to update and distribute the Lagrangian multiplier to the agents. In this paper, we present an alternate approach that *does not require any central coordination.* Hence, it prescribes a truly distributed framework to solve P. In particular, we make use of and generalize the recently developed distributed Lagrangian method in [4, 12] in Section 2, and summarize its convergence properties in Section 3.

Problem P can model the tie-line scheduling problem in electric power systems. To motivate the application, notice that different system operators (SOs) control parts of an interconnected power network and their associated grid assets over a geographical footprint, we call an *area.* Tie-lines are precisely the transmission lines that interconnect different areas. One would ideally solve a joint optimal power flow problem to compute the minimum cost dispatch across the entire power network. Such a solution requires one to aggregate dispatch costs – often collected through bids and offers from a wholesale market organized by the SO – as well as the network parameters within each area. For technical, historical, and legal barriers, such an aggregation is untenable. Distributed approaches are more suited to the task. We remark that tie-lines are generally underutilized today. As is noted in [11], optimal utilization (e.g., using our algorithm) can lead to substantial monetary savings.

We will identify variables x_i as the dispatch within each area that only concerns the SO of area i (denoted henceforth as SO_i). The power flows over the tie-lines connecting areas i and j will be modeled as linear maps of y_i and y_j, where the constraint in (1b) will enforce power flow constraints over the tie-lines. Finally, f_i will model the dispatch cost and $\mathcal{S}_i$ will model the network and generation constraints within area i.

Tie-line schedules are typically fixed with a lead time prior to power delivery. The exact demand and supply conditions are unknown during the scheduling process. In Section 4, we first ignore such uncertainty and reduce the *deterministic tie-line scheduling problem* to an instance of P, and numerically demonstrate our alogrithm on a two area power sytem. Then, we model the uncertainty in demand and supply and formulate the question of *robust tie-line scheduling.* The robust counterpart with the so-called *affine decision rule* approximation, again reduces to an instance of P. The robust problem is motivated by the need to optimize tie-line flows against the uncertainty from variable renewable resources such as wind and solar energy.

1.1 Related literature

Two different threads of the literature inform our work: the theory on distributed algorithms, and the application area of tie-line scheduling in power systems. A distributed approach to solve a joint optimization problem similar to that in P via the so-called *Lagrangian method* can now be found in standard texts, e.g., in [3]. They construct a Lagrangian function for the joint optimization problem, and sequentially update the primal and the dual variables. And, they rely on a central coordinator to communicate the current dual variables, e.g., in [1].

The application of distributed optimization to the tie-line scheduling problem in power systems goes back to the foundational work

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of [7]. In that paper and those that followed, e.g., [8] and the references therein, the authors have adopted the popular Lagrangian approach. Naturally, they rely on a central coordinator to communicate the dual variables to the SOs of each area. Another approach to solve the tie-line scheduling problem is the *primal decomposition* approach. Rather than utilizing a Lagrangian, they directly update two sets of primal variables in a sequential fashion, those that are native to the dispatch within each area and the remainder that define the power flows over the tie-lines. Again, a coordinator mediates between the SOs to update the latter set of variables; see for example [5, 9].

2 THE DISTRIBUTED LAGRANGIAN ALGORITHM

We propose Algorithm 1 to solve P. This algorithm combines a consensus step with a projected distributed sub-gradient algorithm applied to the Lagrangian dual problem of P. To describe the dual problem, we need additional notation. Define $\mathcal{P}^y(\mathcal{S}_i)$ as the projection of $\mathcal{S}_i$ on the y_i coordinates. Then, consider the parametric optimal cost function

$$F_i(y_i) := \underset{x_i}{\text{minimum}}\ f_i(x_i, y_i), \text{ subject to } (x_i, y_i) \in \mathcal{S}_i,$$

for $y_i \in \mathcal{P}^y(\mathcal{S}_i)$, and its *Fenchel conjugate*

$$F_i^*(u) := \underset{y_i \in \mathcal{P}^y(\mathcal{S}_i)}{\text{maximum}} \left\{ u^T y_i - F_i(y_i) \right\}.$$

The dual problem of P is then equivalent to solving

$$\text{DP} \ : \ \underset{\lambda \in \mathbf{R}_+^N}{\text{minimize}} \ \sum_{i=1}^{n} \underbrace{F_i^*(-\lambda^T \mathbf{A}_i) + \lambda^T b_i}_{:= q_i(\lambda)}. \tag{2}$$

Here, $\lambda \in \mathbf{R}_+^N$ is the Lagrange multiplier associated with the coupling constraint (1b). In the sequel, assume $\mathbf{W} \in \mathbf{R}^{n \times n}$ is a *doubly stochastic* matrix[1] with strictly positive diagonals that conforms to the sparsity pattern of $\mathfrak{G}$, i.e., the positive entries of $\mathbf{W}$ define the connectivity of $\mathfrak{G}$. Recall that each agent knows the vector b in (1b). Define $b_i := b/n$ for each $i = 1, \ldots, n$. Finally, let ε be a small positive number, and $\{\alpha(k)\}_{k=0}^{\infty}$ be a nonnegative sequence of step-sizes that parameterizes our algorithm.

The algorithm makes use of the following notation. If a is any vector, a^T denotes its transpose, a^+ denotes its projection on the nonnegative orthant, and $\|a\|_2$ denotes its Euclidean norm.

Remark. *The classical Lagrangian approach requires a central coordinator to maintain and update a global dual multiplier λ. To make the classical approach truly distributed, one can maintain local copies of λ's and leverage an inner consensus loop to compute the global λ. We improve upon such a method by concurrently updating the local copies of the multipliers and the primal variables in steps 5 and 6 in Algorithm 1.*

3 CONVERGENCE PROPERTIES

Our first result finds conditions on the step sizes $\{\alpha(k)\}$ that makes the local copies of the dual variables with each agent converge to an optimizer of the dual problem.

[1] A matrix with nonnegative entries is said to be doubly stochastic if and only if all its rows and columns sum to one.

Algorithm 1 Distributed Lagrangian Method for solving P.

1: **Initialize:**
   $k \leftarrow 1$.
   Agents $i = 1, \ldots, n$ initialize $\lambda_i(1) \in \mathbf{R}_+^N$.
2: **do**
3: For each $i = 1, \ldots, n$, agent i executes:
4: Communicate with neighbors to compute
$$v_i(k) \leftarrow \sum_{j=1}^{n} W_{ij} \lambda_j(k).$$
5: $(x_i(k+1), y_i(k+1))$
$$\leftarrow \underset{(x_i, y_i) \in \mathcal{S}_i}{\text{argmin}} \ [f_i(x_i, y_i) + v_i^T(k)(\mathbf{A}_i y_i - b_i)].$$
6: $\lambda_i(k+1) \leftarrow [v_i(k) + \alpha(k)(\mathbf{A}_i y_i(k+1) - b_i)]^+$.
7: $k \leftarrow k + 1$.
8: **while** $\|\lambda_i(k) - v_i(k)\|_2 > \varepsilon$.

Theorem 3.1. *Suppose Assumption 1 holds. If the nonincreasing nonnegative sequence $\{\alpha(k)\}$ satisfies*

$$\sum_{k=1}^{\infty} \alpha(k) = \infty, \quad \textit{and} \quad \sum_{k=1}^{\infty} \alpha^2(k) < \infty, \tag{3}$$

then $\{x_i(k), y_i(k), \lambda_i(k)\}$ from Algorithm 1 satisfies

(a) $\lim_{k\to\infty} \lambda_i(k) = \lambda^*$ is an optimizer of DP *for each $i = 1, \ldots, n$,*
(b) $\lim_{k\to\infty} \sum_{i=1}^{n} f_i(x_i(k), y_i(k))$ is the optimal value of P.

One can choose $\alpha(k) = 1/k$ to satisfy (3). To present our next result, let $q(\lambda) = \sum_{i=1}^{n} q_i(\lambda)$, and q^* denote the value of q at an optimal solution of DP. The next result shows that q evaluated at a time-averaged α-weighted local copies of the dual variables (with each agent) converges to the optimal value q^* for a particular choice of step-sizes. And, the difference of q at this 'average' λ from q^* scales as $O\left(\ln(k)/\sqrt{k}\right)$.

Theorem 3.2. *Suppose Assumption 1 holds. If $\alpha(k) = 1/\sqrt{k}$ for $k \geq 1$, then $\{\lambda_i(k)\}$ from Algorithm 1 satisfies*

$$q(z_i(k)) - q^* \leq \frac{n}{\sqrt{k}} \left(\psi_1 \ln(k) + \frac{\psi_2 n + \psi_3 \sqrt{n} \ln(k)}{1 - \sigma_2(\mathbf{W})} \right), \tag{4}$$

for positive constants ψ_1, ψ_2, ψ_3, where $\sigma_2(\mathbf{W})$ denotes the second largest singular value of $\mathbf{W}$, and $z_i(k) := \sum_{\ell=1}^{k} \alpha(\ell)\lambda_i(\ell) / \sum_{\ell=1}^{k} \alpha(\ell)$ for each $i = 1, \ldots, n$.

4 TIE-LINE SCHEDULING IN MULTI-AREA POWER SYSTEMS

In this section, we apply the distributed Lagrangian method to the tie-line scheduling problem. We only present the case for $n = 2$ areas. Generalization to $n > 2$ areas is straightforward.

The deterministic problem with known net demands. Consider two areas, labelled 1 and 2. For each $i = 1, 2$, let the power network in area i be comprised of N_i internal buses, and $N_{\bar{i}}$ boundary buses. The internal buses (nodes) are the ones that do not have tie-lines connected to them; the boundary buses (nodes) are the ones that do. Assume that each internal bus has a dispatchable generation resource that supplies $g_i \in \mathbf{R}^{N_i}$. Each internal bus also has a net demand resource. They draw $\xi_i \in \mathbf{R}^{N_i}$ across the network.

Assume that the boundary buses do not have any generation or net demand. Adopting a linear dispatch cost structure for each area the deterministic tie-line scheduling problem is given by

$$\underset{\substack{g_i,\theta_i,\theta_{\bar{i}}\\ i=1,2}}{\text{minimize}} \quad \sum_{i=1,2} c_i^T g_i,$$

$$\text{subject to} \quad \begin{pmatrix} \mathbf{B}_{11} & \mathbf{B}_{1\bar{1}} & 0 & 0 \\ \mathbf{B}_{\bar{1}1} & \mathbf{B}_{\bar{1}\bar{1}} & \mathbf{B}_{\bar{1}\bar{2}} & 0 \\ 0 & \mathbf{B}_{\bar{2}1} & \mathbf{B}_{\bar{2}\bar{2}} & \mathbf{B}_{\bar{2}2} \\ 0 & 0 & \mathbf{B}_{2\bar{2}} & \mathbf{B}_{22} \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_{\bar{1}} \\ \theta_{\bar{2}} \\ \theta_2 \end{pmatrix} = \begin{pmatrix} g_1 - \xi_1 \\ 0 \\ 0 \\ g_2 - \xi_2 \end{pmatrix}, \tag{5a}$$

$$\mathbf{H}_i\theta_i + \mathbf{H}_{\bar{i}}\theta_{\bar{i}} \le L_i, \ i = 1,2, \tag{5b}$$

$$\mathbf{H}_{12}\theta_{\bar{1}} + \mathbf{H}_{21}\theta_{\bar{2}} \le L_{12}, \tag{5c}$$

$$\underline{g}_i \le g_i \le \overline{g}_i, \ i = 1,2. \tag{5d}$$

In (5a), $\theta_i \in \mathbf{R}^{N_i}$ and $\theta_{\bar{i}} \in \mathbf{R}^{N_{\bar{i}}}$ denote the vectors of voltage phase angles at the internal and boundary buses in area i, respectively Collectively, (5a) – (5c) model the Kirchhoff's laws and the transmission constraints of the power networks in the two areas, together. The linear power flow model adopted here can be obtained from the so-called *DC approximations* of the Kirchhoff's laws [10]. The generation constraints at internal buses are modeled in (5d). Using $x_i = \left(g_i^T,\ \theta_i^T\right)^T$ and $y_i = \left(\theta_{\bar{1}}^T,\ \theta_{\bar{2}}^T\right)^T$, $i = 1,2$, (5) can be written as

$$\underset{\substack{x_i,y_i\\ i=1,2}}{\text{minimize}} \quad \sum_{i=1,2} \tilde{c}_i^T x_i,$$

$$\text{subject to} \quad \mathbf{S}_i^x x_i + \mathbf{S}_i^y y_i + \mathbf{S}_i^\xi \xi_i \le d_i, \ i = 1,2, \tag{6a}$$

$$y_1 - y_2 = 0, \quad y_1, y_2 \in \mathcal{Y}, \tag{6b}$$

for suitably defined $\tilde{c}_i, \mathbf{S}_i^x, \mathbf{S}_i^y, \mathbf{S}_i^\xi, d_i$ for $i = 1,2$, and $\mathcal{Y}$. Details are omitted for brevity. Problem (6) is an instance of P.

Numerical experiment. We adopt a case study from [6] that considers the IEEE 14 and 30 bus systems to represent the two areas. In Figure 1, we plot $q_1(\lambda_1(k)) + q_2(\lambda_2(k))$ as a function of k from Algorithm 1 applied to (6). With a step-size of $\alpha(k) = 1/k$, it converges to negative of the optimal aggregate cost. We further capture the effect of altering the step sizes by varying s for $\alpha(k) = 1/k^s$. As s increases, the algorithm converges faster.

The (adjustable) robust variant with uncertain net demand. We next present the case where tie-line flows are optimized to minimize the maximum aggregate dispatch costs across the areas. The maximum cost is computed by varying the net demand ξ_i over the polytope

$$\Xi_i := \{\xi_i \in \mathbf{R}^{N_i} \ : \ \mathbf{D}_i\xi_i \le d_i\}, \text{ for } i = 1,2.$$

Recall that $y_i \in \mathbf{R}^{N_{\bar{1}}+N_{\bar{2}}}$ equals the vector of voltage phase angles at the boundary buses. To fix the tie-line flow schedule, we therefore assume that y_i's are fixed prior to the time of power delivery, and allow x_i's to be square-integrable maps of y_i and ξ_i. Searching over square-integrable maps can be challenging. Hence, we restrict the search over *affine decision rules* of the form $x_i(y_i, \xi_i) = x_i^y + \mathbf{X}_i^\xi \xi_i$, and optimize over x_i^y and $\mathbf{X}_i^\xi$. With this approximation, the robust tie-line scheduling problem becomes

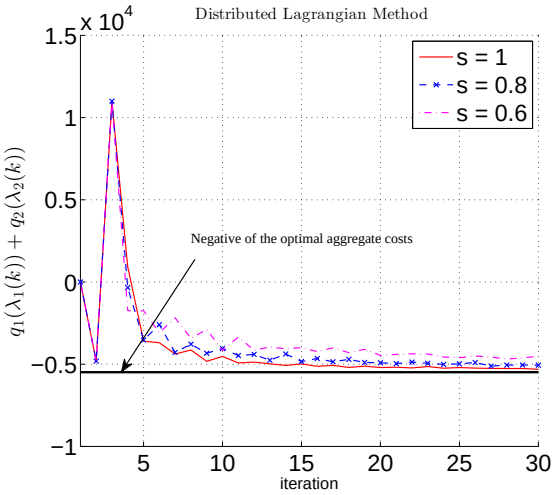

Figure 1: Illustration of Algorithm 1 on (6).

$$\underset{\substack{x_i^y,\mathbf{X}_i^\xi,y_i\\ i=1,2}}{\text{minimize}} \quad \underset{\substack{\xi_i\in\Xi_i,\\ i=1,2}}{\text{maximum}} \ \sum_{i=1}^{2} t_i,$$

$$\text{subject to} \quad \tilde{c}_i^T \left[x_i^y + \mathbf{X}_i^\xi \xi_i\right] \le t_i, \tag{7a}$$

$$\mathbf{S}_i^x \left[x_i^y + \mathbf{X}_i^\xi \xi_i\right] + \mathbf{S}_i^y y_i + \mathbf{S}_i^\xi \xi_i \le s_i, \tag{7b}$$

$$\text{for all } \xi_i \in \Xi_i, \ i = 1,2,$$

$$y_1 - y_2 = 0, \quad y_1, y_2 \in \mathcal{Y}. \tag{7c}$$

Robust enforcement of the constraints in (7a) and (7b) render the problem *semi-infinite*. Duality theory of linear programming can be leveraged to reduce (7) into a standard linear program that is an instance of P, e.g., see [2]. We omit the details for brevity.

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