Load-side Frequency Regulation with Limited Control Coverage

Extended Abstract

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ABSTRACT

Increasing renewable energy increases uncertainty in energy systems. As a consequence, generator-side control for frequency regulation, impacted by the slow reaction of generators to meet urgent needs, may no longer suffice. With increasing integration of smart appliances which are able to sense, communicate and control, load-side control can help alleviate the aforementioned problem as it reacts fast and helps to localize disturbances. However, almost all existing methods for optimal load-side control require full information control coverage in the system. Framing the problem as an optimization problem and applying saddle-point dynamics, we obtain a control law that rebalances power and asymptotically stabilizes frequency after a disturbance. We generalize previous work to design a controller which only requires partial control coverage over all nodes, yet still achieves secondary frequency control. We verify these results via simulation.

1 INTRODUCTION

Frequency regulation aims to keep frequency of a power system close to its nominal value. Frequency deviates with demand deviations, and propagates through the network, leading to potential blackouts. Previously, generator-side control suffices for frequency regulation, and is typically classified into three stages: (i) Primary - stabilization, (ii) Secondary - fast, expensive recovery and (iii) Tertiary - slow, cheaper recovery. As renewables become increasingly utilized, demand fluctuates more due to the lack of predictability in yield of renewables. This problem will continue to grow as states aim for 50% renewables by 2030 [9], and also since alternatives, e.g. battery storage programs, are cost-ineffective, with break-even costs estimated to be \$197/kW [6].

Controllable loads, e.g. smart household appliances, have the ability to decrease energy imbalance, with simulations and real-world demonstrations highlighting their potential [4]. However, in a large scale grid, not every node have controllable loads capable of supporting frequency regulation, and therefore *it is important to consider situations where the load control coverage is not full*.

1.1 Literature Review

The use of primal-dual dynamics for control started out with Kelly [5] and Low [7] in network resources and [3] for stability of primal-dual dynamics. It has been recently used in the power systems community for frequency control by [13] with prior work [14] identifying power system swing equations as a primal-dual dynamic, setting the foundation for a series of work in this area (e.g. [8, 12]). There is a larger body of work on distributed frequency control (see e.g. [11] and references therein for recent examples) that does not employ the primal-dual framework.

1.2 Summary

In this work, we focus on load-side participation for secondary frequency control. In particular, assuming that the power or frequency imbalance can be reported through a distress signal in real-time, we show that a limited coverage of nodes participating in load control can cooperate to yield secondary frequency regulation. Simulation is performed on a non-linear system, demonstrating the effectiveness of our control. Our contributions can be summarized as follows:

(1) Primary Regulation under limited control coverage.
(2) Secondary Regulation under perfect disruption prediction.
(3) Bounding the gap between nominal frequencies and stabilized frequencies under errors in such predictions.

2 NETWORK MODEL & PRELIMINARIES

We adopt the model of Zhao et al. [13]. The power network is described by a graph $\mathscr{G} = (N, E)$ where $N = \{1, \dots, |N|\}$ is the set of buses and $E \subset N \times N$ is the set of transmission lines (i, j), such that if $(i, j) \in E$, then $(j, i) \notin E$. We also consider a communication graph $\mathscr{G}'_A$ with edges linking a subset of buses A to their neighbors on the power network. This is a subgraph $\mathscr{G}'_A = (A, E')$ induced by the set of nodes $A \subseteq N$. We partition the buses as $N = G \cup L$ and use G and L to denote the set of generator and load buses respectively. Generator nodes $i \in G$ generates electrical power and may also have loads. Load nodes $i \in L$ have only loads. The subset $A \subseteq N$ corresponds to active loads, i.e. nodes with load control, and requires communication with their neighbors. We identify the set of nodes with frequency sensitive load with F.

As in [13], we have that the power system dynamics as follows:

$$M_i \dot{\omega}_i = P_i^m - (\mathbf{1}_{i \in A} d_i + \mathbf{1}_{i \in F} \hat{d}_i) - \sum_{e \in E} C_{ie} P_e, \qquad \forall i \in G \quad \text{(1a)}$$

$$0 = P_i^m - (\mathbf{1}_{i \in A} d_i + \mathbf{1}_{i \in F} \hat{d}_i) - \sum_{e \in E} C_{ie} P_e, \qquad \forall i \in L \quad \text{(1b)}$$

$$\dot{P}_{ij} = B_{ij}(\omega_i - \omega_j), \qquad \forall (i, j) \in E \quad \text{(1c)}$$

where d_i denotes an aggregate controllable load and $\hat{d} := D_i \omega_i$ denotes an aggregate uncontrollable but frequency-sensitive load and M_i is the generator's inertia. P_i^m is the mechanical power generated or used and P_{ij} is the real power flow from i to j. C_{ie} are the elements of the incidence matrix $C \in \mathbb{R}^{|N| \times |E|}$ of the graph $\mathscr{G}$ defined as $C_{ie} = -1$ if $e = (j, i) \in E$, $C_{ie} = 1$ if $e = (i, j) \in E$ and $C_{ie} = 0$ otherwise. The incidence matrix C' for the communication graph is similarly defined. We refer the reader to Zhao et al. [13] for a detailed motivation of the model. We are interested in the situation where the system is originally at an equilibrium, i.e. when $\dot{\omega} = \dot{P}_{ij} = 0$, and then the system is perturbed locally.

3 STABILIZATION UNDER LIMITED CONTROL COVERAGE

In this section, we design a distributed control mechanism that rebalances the system while driving the frequency back to its nominal value, even when not all nodes participate in load control.

We define an optimal load control (OLC) problem as follows:

$$\min_{d,\hat{d},P,R} \quad \sum_{i\in A} c_i(d_i) + \sum_{i\in F} \frac{\hat{d}_i^2}{2D_i}$$

$$\text{s.t.} \quad P_i^m - (\mathbf{1}_{i\in A} d_i + \mathbf{1}_{i\in F} \hat{d}_i) = \sum_{e\in E} C_{ie} P_e, \quad \forall i \in N \tag{2a}$$

$$P_i^m + \hat{P}_i - d_i = \sum_{e\in E'} C'_{ie} R_e, \quad \forall i \in A \tag{2b}$$

Unlike [8] and [13], *we do not assume that the load control coverage is full, nor do we assume that every node is communicable and admit frequency sensitive loads*. $\hat{P}_i$ is a constant which serves as a prediction to the disruption in power[1] and R_e is a virtual line flow. We make the following assumptions on the OLC, i.e. (i) cost functions c_i are strictly convex and twice continuously differentiable; and (ii) OLC is feasible, implying Slater's condition since constraints in OLC are linear [1].

Let ν_i be the Lagrange multiplier associated with (2a) and λ_i for (2b). The dual of this problem can then be written as:

$$\Phi(\nu,\lambda,d,\hat{d},P,R) = \max_{\nu,\lambda} \min_{d,\hat{d},P,R} \mathscr{L}(d,\hat{d},P,R,\nu,\lambda) \tag{3}$$

Differentiating with respect to d_i and $\hat{d}_i$, the minimizer satisfies:

$$c'_i(d_i) = \nu_i + \lambda_i, \ \forall i \in A \text{ and } \hat{d}_i = D_i \nu_i, \ \forall i \in F \tag{4}$$

which suggests that controllable loads should be set via:

$$d_i(\nu_i,\lambda_i) = (c'_i)^{-1}(\nu_i + \lambda_i) \tag{5}$$

The maximum in (3) can only be attained if $\nu_i = \nu_j$ for all $i, j \in N$ and $\lambda_i = \lambda_j$ for all $i, j \in C'_k$ where C'_k is a connected component of the virtual network. Substituting these back into (3) implies that the dual of OLC can be equivalently written as the maximization of:

$$\Phi(\nu,\lambda) = \sum_{i\in N} \nu_i P_i^m + \mathbf{1}_{i\in A}(c_i(d_i(\nu_i,\lambda_i)) - \nu_i d_i(\nu_i,\lambda_i)) - \mathbf{1}_{i\in F}\left(\frac{D_i \nu_i^2}{2}\right) + \mathbf{1}_{i\in A}\left(\lambda_i(P_i^m + \hat{P}_i - d_i(\nu_i,\lambda_i))\right) \tag{6}$$

Lemma 3.1. *The function in* (6) *is separable, i.e.* $\Phi(\nu,\lambda) = \sum_{i\in N} \Phi_i(\nu_i,\lambda_i)$*, where:*

$$\Phi_i(\nu_i,\lambda_i) = \nu_i P_i^m + \mathbf{1}_{i\in A}(c_i(d_i(\nu_i,\lambda_i)) - \nu_i d_i(\nu_i,\lambda_i)) - \mathbf{1}_{i\in F}\frac{D_i\nu_i^2}{2} + \mathbf{1}_{i\in A}(\lambda_i(P_i^m + \hat{P}_i - d_i(\nu_i,\lambda_i)))$$

Additionally, the Hessian of Φ_i *is given by:*

$$H_i = \begin{bmatrix} \mathbf{1}_{i\in A}(-d'_i) + \mathbf{1}_{i\in F}(-D_i) & \mathbf{1}_{i\in A}(-d'_i) \\ \mathbf{1}_{i\in A}(-d'_i) & \mathbf{1}_{i\in A}(-d'_i) \end{bmatrix}$$

where $d'_i = \frac{\partial d_i}{\partial \nu_i} = \frac{\partial d_i}{\partial \lambda_i}$.

Therefore *strict concavity holds only when node* $i \in A \cap F$, as in [13]. If $i \in A$ (resp. $i \in A \cup F$), Φ_i strictly concave in λ_i (resp. ν_i).

[1] This idea is due to Lachlan Andrew

Lemma 3.2. *Given a connected graph* $\mathscr{G} = (N,E)$ *and a communication graph* $\mathscr{G}$*, fix the constants* $\hat{P}_i$*. Then there exist scalars* ν^*, λ_k^**,* $\forall k$ *(where* C'_k *is a connected component of the graph* $\mathscr{G}'$*) such that* $(d^*, \hat{d}^*, P^*, R^*, \nu^*, \lambda^*)$ *is primal-dual optimal for OLC and its dual if and only if* $(d^*, \hat{d}^*, P^*, R^*)$ *is primal feasible,* (ν^*, λ^*) *is dual feasible, and* (4) *and* (5) *hold.*

In addition, if we set $\hat{P}_i = \frac{\sum_{i\in(N-A)} P_i^m}{|A|}$ *in (2b), we have that* $\nu^* = 0$*. In particular, secondary frequency control is attained.*

In the case where the prediction in the disruption of supply is inaccurate, frequency stabilizes and we can bound the frequency deviation with respect to the error of prediction:

Lemma 3.3. *Suppose* $|\sum_{i\in A} \hat{P}_i - \sum_{i\in(N-A)} P_i^m| < \varepsilon$*, then*

$$\left|\sum_{i\in F} \hat{d}_i^*\right| < \varepsilon, \text{ and, therefore, } |\nu^*| < \frac{\varepsilon}{\sum_{i\in F} D_i}$$

In our setting of $\hat{P}_i$, we assume perfect prediction, and apply a "fair" distribution of the information, which (intuitively from (5)) affects the amount of load control and can be done more optimally in the presence of constraints on lines or controllable loads.

4 DISTRIBUTED OPTIMAL LOAD CONTROL

We focus on the case when we set $\hat{P}_i = \frac{\sum_{i\in(N-A)} P_i^m}{|A|}$, and propose a distributed optimal control mechanism. We will assume that $F \neq \emptyset$.

Let

$$\begin{aligned}\mathscr{L}(P,R,\nu,\lambda) &= \min_{d,\hat{d}} \mathscr{L}(d,\hat{d},P,R,\nu,\lambda) \\ &= \mathscr{L}(d(\nu,\lambda),\hat{d}(\nu),P,R,\nu,\lambda) \\ &= \Phi(\nu,\lambda) - \nu^T C P - \lambda^T C' R\end{aligned}$$

where $\mathscr{L}(d,\hat{d},P,R,\nu,\lambda)$ as defined in (6), and $d(\nu,\lambda)$, $\hat{d}(\nu)$ as defined in (4). We propose the following partial primal-dual algorithm:

$$\dot{\nu}_i = \zeta_i \left[P_i^m - \mathbf{1}_{i\in A} d_i - \mathbf{1}_{i\in F} D_i \nu_i - \sum_{e\in E} C_{ie} P_e\right], \ i \in G \tag{7a}$$

$$0 = P_i^m - \mathbf{1}_{i\in A} d_i - \mathbf{1}_{i\in F} D_i \nu_i - \sum_{e\in E} C_{ie} P_e, \ i \in L \tag{7b}$$

$$\dot{\lambda}_i = \eta_i \left[P_i^m + \hat{P}_i - d_i - \sum_{e\in E'} C'_{ie} R_e\right], \ i \in A \tag{7c}$$

$$\dot{P}_{ij} = \beta_{ij}(\nu_i - \nu_j), \qquad \dot{R}_{ij} = \alpha_{ij}(\lambda_i - \lambda_j) \tag{7d}$$

where part of the dynamics is a gradient descent step as in the primal-dual algorithm. Identifying ζ_i with M_i^{-1}, β_{ij} with B_{ij}, the power system dynamics in (1) can be interpreted as a subset of the dynamics above, and we can identify the Lagrange multiplier ν_i with ω_i, with parameters η_i and α_{ij} determining how much weight we place on information from the virtual variables.

Consider the Lagrangian of the dual of the OLC problem:

$$\mathscr{L}_D(\nu_G,\nu_L,\lambda,\pi) := \sum_{i\in N} \Phi_i(\nu_i,\lambda_i) - \sum_{(i,j)\in E} \pi_{ij}^N(\nu_i - \nu_j) - \sum_{(i,j)\in E'} \pi_{ij}^C(\lambda_i - \lambda_j)$$

To amend for the partiality of the primal-dual dynamics we used, consider the partial Lagrangian:

$$\mathscr{L}_D(P,R,\nu_G,\lambda) = \max_{\nu_L} \mathscr{L}_D(P,R,\nu_G,\nu_L,\lambda) \quad (8)$$

and by considering the same constraints as before, ν_L takes on its unique maximizer value. By using the Envelope Theorem, we can then compute the partial derivatives of (8) with respect to the variables (P,R,ν_G,λ), and end up with the same equations as (7). By the following lemma[2], we then know the proposed dynamics is asymptotically stable.

LEMMA 4.1. *For a C^1 function $F(x,z): \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$, if*

(1) *F is globally convex in x and linear in z,*
(2) *for each $(x_*,z_*) \in Saddle(F)$, if $F(x,z_*) = F(x_*,z_*)$, then $F(x,z_*) \in Saddle(F)$,*

then $Saddle(F)$ is globally asymptotically stable under the saddle point dynamics and convergence of trajectories is to a point.

With the above results, we state the main theorem of this work:

THEOREM 4.2. *Assume the physical graph $\mathscr{G} = (N,E)$ is connected, $A \neq \emptyset$, $F \neq \emptyset$, and $L \subseteq F \cup A$. The system with control steps as defined in (7) asymptotically converges to an equilibrium point where primary frequency control is attained.*

If in addition, an exact estimate $\hat{P}_i$ of the power step change is available, secondary frequency control can be attained.

In the case of inaccurate predictions of the power step change, the performance of the dynamics can be bounded as:

$$|\nu^*| < \frac{\varepsilon}{\sum_{i \in F} D_i}$$

where ε is the error in prediction.

5 NUMERICAL ILLUSTRATIONS

To demonstrate the performance of our load control to aid secondary control, we apply our control on the IEEE 39-bus test system. Unlike the analyzed linear model, the simulation adopts a nonlinear set-up, including e.g. nonlinear governor dynamics and power flows. We pick out a subset of nodes randomly for load control, and choose one node randomly to add a step increase of 1pu (based on 100MVA) of its current load. We do not limit the amount of load control the nodes can utilize, and illustrate that secondary frequency control is attained. The results are illustrated in Figure 1.

6 CONCLUSION

In this work, we propose a control design for frequency control based on a linearized model of the swing equations. By exploiting the amenability of the primal-dual dynamics, part of the dynamics can be designed to match power system dynamics. The control steps and virtual variables are updated based on the values of the physical variables. We prove that under some mild conditions, the system provably converges with an accurate prediction of power disruption, and bounds on the performances are provided with respect to inaccuracies in predictions. We implement our control steps on a non-linear simulation, demonstrating the performance of our control.

A full version of this work can be found in [10].

[2] C^1 functions are continuously differentiable functions and $Saddle(F)$ are the sets of saddle points of the function F.

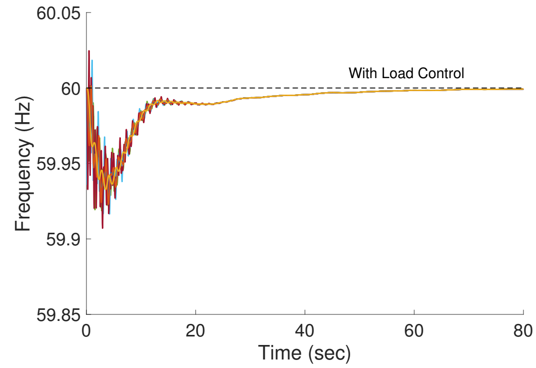

Figure 1: Dynamics of a perturbed system with proposed load control. Four nodes (1, 3, 30 and 39) chosen to be controllable and one node (35, located far away) experiences disruption. These nodes are chosen to explicitly illustrate the ability to perform secondary frequency control when the load control coverage is small and are not close to the disrupted node.

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