

Do we need two forms of feedback in the Rate Control Protocol (RCP)?

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ABSTRACT

There is considerable interest in the networking community in explicit congestion control as it may allow the design of a fair, stable, low loss, low delay, and high utilization network. The Rate Control Protocol (RCP) is an example of such a congestion control protocol. The current design of RCP suggests that it should employ two forms of feedback; i.e. rate mismatch and queue size, in order to manage its flow control algorithms. An outstanding design question in RCP is whether the presence of queue size feedback is useful, given the presence of feedback based on rate mismatch. To address this question, we conduct analysis (stability and Hopf bifurcation) and packet-level simulations. The analytical results reveal that the presence of queue size feedback in the protocol specification may induce a sub-critical Hopf bifurcation, which can lead to undesirable system behavior. The analysis is corroborated by numerical computations and some packet-level simulations. Based on our work, the suggestion for RCP is to only include feedback based on rate mismatch in the design of the protocol.

1. INTRODUCTION

The Transmission Control Protocol (TCP) is the most commonly used transport layer protocol, which handles congestion control in the Internet. Currently, TCP uses packet loss and delay as signals of network congestion. Delay and loss are damage to packets, and hence this implicit signaling will impact performance limits. This inherent limitation has led to the development of congestion control protocols that could utilize *explicit* feedback. In the class of algorithms that employ explicit feedback, the Rate Control Protocol (RCP) [2, 4, 7] has the potential to offer a fair, stable, low-loss, low-delay network operating at a high link utilization. Moreover, RCP continues to receive attention not only in the currently used host-centric (IP-based) networks [3, 7, 12], but also in future data-centric networking architectures like Named Data Networking (NDN) [8, 9, 11]. See [15] for an overview of Named Data Networking. In NDN, there is no IP address, and all data packets are named with unique names. Moreover, the data can be fetched from multiple sources via multiple paths which makes the implicit signal-

ing mechanism used by TCP unreliable in NDN [11]. By using explicit feedback, congestion control protocols may also greatly reduce flow completion times [4]. RCP computes the fair rate using two forms of feedback: rate mismatch and queue size. From an engineering perspective, two key design issues in RCP are the following: (i) how new flows may be admitted at a fair and high starting rate while maintaining small queues, and (ii) whether the presence of both forms of feedback is desirable for managing flow control. The issue of admission management of new flows in RCP was initially addressed in [7]. This paper addresses the design question related to the presence of two forms of feedback.

Most queuing systems routinely share information regarding waiting times, or queue lengths, with end systems. Such information can certainly influence the behavior of end systems. If such feedback is not instantaneous but is time-delayed, it can have a significant impact on the stability and dynamics of the underlying system. In RCP, the feedback from routers to end-systems is time-delayed. Thus, we address the impact of having two forms of feedback on the stability and dynamical properties of RCP. In [7], a local stability analysis was conducted for RCP, but such an analysis was not able to offer any design insights on the use of two forms of feedback. This provides motivation for some non-linear analysis, in order to study some additional dynamical properties of the RCP system. We employ a bifurcation theoretic style of analysis where we study the dynamics of the system as it transits from a stable to an unstable regime. We conduct our analysis on a proportionally fair variant of RCP [7]. Analytical results reveal that the presence of queue feedback in RCP can induce a sub-critical Hopf bifurcation, for some parameter values. A sub-critical Hopf bifurcation can result either in large amplitude limit cycles or unstable limit cycles [13], and hence should be avoided in engineering applications. Our study suggests that the presence of both forms of feedback may in fact be detrimental to the performance of RCP. The analysis is complemented with numerical computations and some packet-level simulations.

2. MODEL

For our analysis, the setting that we consider is a single bottleneck link of capacity C , carrying flows with a same round-trip time τ . To conduct a unified analysis of both the design choices, we introduce an exogenous non-dimensional parameter $\kappa > 0$ which acts as the bifurcation parameter. In this setting, the non-linear model of a proportionally fair variant of RCP [7] is

$$\frac{d}{dt}R(t) = \frac{\kappa a R(t)}{C\tau} \left(C - y(t) - bCp(y(t)) \right), \quad (1)$$

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where $y(t) = R(t - \tau)$ is the aggregate load, and $p(y) = y/(2(C - y))$ is the *mean* queue size. Here, $R(t)$ denotes the fair share rate, a and b are non-negative protocol parameters. To model RCP which uses only rate mismatch feedback, b is set to zero in (1). To aim for a particular target link utilization, say a fraction γ of the actual link capacity, C is replaced with γC . Then, the system model is

$$\frac{d}{dt}R(t) = \frac{\kappa a R(t)}{\gamma C \tau} (\gamma C - R(t - \tau)). \quad (2)$$

3. STABILITY ANALYSIS

We outline some results for the local stability of RCP.

3.1 With queue feedback

Let $R(t) = R^* + u(t)$, and linearizing (1) about its equilibrium yields

$$\frac{d}{dt}u(t) = -(\kappa a(1 + \rho^*)/\tau) u(t - \tau), \quad (3)$$

where $\rho^* = R^*/C = (b + 4 - \sqrt{b^2 + 8b})/4$ is the equilibrium link utilization, and R^* denotes the equilibrium rate. Using results from [10], we obtain the necessary and sufficient condition for local stability of (1) as

$$\kappa a(1 + \rho^*) < \pi/2, \quad (4)$$

and the first Hopf bifurcation occurs at $\kappa a(1 + \rho^*) = \pi/2$.

3.2 Without queue feedback

The equilibrium of (2) is $R^* = \gamma C$. Linearizing (2) about R^* , we obtain

$$\frac{d}{dt}u(t) = -(\kappa a/\tau) u(t - \tau). \quad (5)$$

From [10], the necessary and sufficient condition for local stability of (2) can be written as

$$\kappa a < \pi/2. \quad (6)$$

and the first Hopf bifurcation occurs at $\kappa a = \pi/2$.

We now conduct a Hopf bifurcation analysis to gain some insights into the system behavior in the unstable regime.

4. HOPF BIFURCATION ANALYSIS

In this section, we analyze the nature of the Hopf bifurcation of RCP in the presence and absence of queue feedback.

4.1 With queue feedback

To analyze the type of the Hopf bifurcation in RCP, we employ the analysis and results obtained in [10] for a non-linear delay differential equation. The analytical tools employed to determine the type of the Hopf bifurcation are Poincaré normal forms and the center manifold theorem [6, 10]. We first start by noting that the system we consider is a special case of the following non-linear delay differential equation

$$\frac{d}{dt}x(t) = \kappa f(x(t), x(t - \tau)) \quad (7)$$

where f has a unique equilibrium denoted by (x^*, y^*) .

The Taylor series expansion of (1) about its equilibrium, up to the third order terms, is

$$\begin{aligned} \frac{d}{dt}u(t) = & \kappa(\xi_y u(t - \tau) + \xi_{xy} u(t)u(t - \tau) + \xi_{yy} u^2(t - \tau) \\ & + \xi_{xyy} u(t)u^2(t - \tau) + \xi_{yyy} u^3(t - \tau)) \end{aligned} \quad (8)$$

where, letting f^* denote the evaluation of f at its equilibrium

$$\begin{aligned} \xi_y &= f_y^* = \frac{-a(1 + \rho^*)}{\tau}, \quad \xi_{xy} = f_{xy}^* = \frac{-a(1 + \rho^*)}{C\tau\rho^*}, \\ \xi_{yy} &= \frac{f_{yy}^*}{2} = \frac{-a}{C\tau(1 - \rho^*)}, \quad \xi_{xyy} = \frac{f_{xyy}^*}{2} = \frac{-a}{C^2\tau\rho^*(1 - \rho^*)}, \\ \xi_{yyy} &= \frac{f_{yyy}^*}{6} = \frac{-a}{C^2\tau(1 - \rho^*)^2}. \end{aligned}$$

Following the style of analysis in [10], one can get the expression for the Lyapunov coefficient μ_2 associated with (8):

$$\begin{aligned} \mu_2 = & \xi_{xy}^2 \frac{3\pi - 2}{5\pi\xi_y^2} + \xi_{yy}^2 \frac{2(11\pi - 4)}{5\pi\xi_y^2} + \xi_{xy}\xi_{yy} \frac{(7\pi - 18)}{5\pi\xi_y^2} \\ & + \frac{2\xi_{xyy}}{\pi\xi_y} - \frac{3\xi_{yyy}}{\xi_y}. \end{aligned} \quad (9)$$

After substituting the values of the Taylor series coefficients in (9), and simplifying, we get

$$\begin{aligned} \mu_2 = & \frac{1}{C^2 5\pi\rho^{*2}(1 - \rho^{*2})^2} \left((3\pi - 2)\rho^{*4} - (22\pi - 8)\rho^{*3} \right. \\ & \left. - (4 - \pi)\rho^{*2} + (7\pi - 8)\rho^* + (3\pi - 2) \right), \end{aligned} \quad (10)$$

where the sign of μ_2 determines the type of the Hopf bifurcation. If $\mu_2 > 0$ ($\mu_2 < 0$), then the Hopf bifurcation is *super-critical* (*sub-critical*). From (10), we can deduce that only the numerator terms determine the sign of μ_2 . Hence we plot the variation in the numerator of μ_2 as ρ^* is varied from 0 to 1. We observe from Figure 1 that μ_2 reaches zero at $\rho^* = 0.6621$, and hence the type of the Hopf bifurcation changes from super-critical to sub-critical after $\rho^* = 0.6621$. We now validate this using some numerical examples.

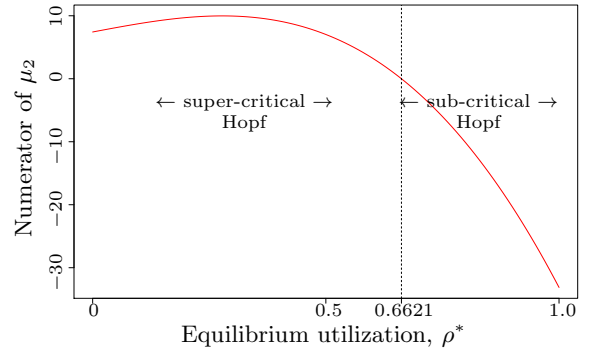


Figure 1: In the case of RCP with queue feedback, the Hopf bifurcation is super-critical for $\rho^* < 0.6621$ and sub-critical for $\rho^* > 0.6621$.

Numerical Examples: Let us consider the RCP system with $C = 10$ and $\tau = 100$. For $\rho^* = 0.55$ ($b = 0.736$) and $a = 1.0$, the system undergoes a Hopf bifurcation at $\kappa = \kappa_c = 1$. From Figure 1, we can observe that $\mu_2 > 0$ (super-critical) for $\rho^* = 0.55$. Whereas, for $a = 0.827$ and $\rho^* = 0.9$ ($b = 0.022$), we have $\mu_2 < 0$ (sub-critical). To validate this, numerical solutions obtained using XPPAUT [5] are shown in Figure 2. For $\kappa = 1.05 > \kappa_c$ i.e. after the bifurcation point, the system gives rise to stable limit cycles

when $\rho^* = 0.55$. Whereas for $\rho^* = 0.9$, the limit cycles are unstable and the solution would eventually blow up.

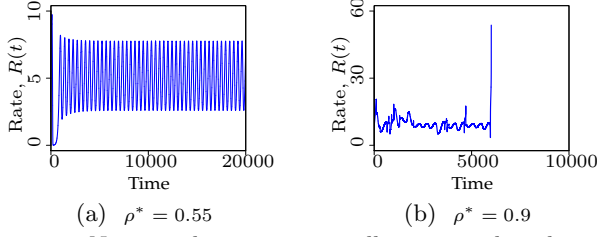


Figure 2: Numerical computations illustrating that the system which includes queue feedback exhibits a super-critical Hopf for $\rho^* = 0.55$, and a sub-critical Hopf for $\rho^* = 0.9$.

4.2 Without queue feedback

In [14], it has been shown that (2) undergoes the first local Hopf bifurcation at $\kappa = \kappa_c$, where $\kappa_c a = \pi/2$. If the Hopf condition is just violated, the system would lose local stability via a super-critical Hopf bifurcation and the amplitude of the bifurcating limit cycles will be proportional to

$$R^* \sqrt{\frac{20\pi(\kappa - \kappa_c)}{3\pi - 2}}. \quad (11)$$

Discussion: The analytical results reveal that the presence of queue feedback in RCP can induce a sub-critical Hopf bifurcation, at high link utilization. A sub-critical Hopf would lead either to unstable limit cycles, or to limit cycles with a very large amplitude. Either of these outcomes is detrimental to system performance and is undesirable in engineering applications. Hence, it is advisable to go with the design choice that uses only rate mismatch feedback. See [1] for an extended version of this paper.

5. PACKET-LEVEL SIMULATIONS

For our simulations, we consider the following set up: $C = 1$ Gb/s, number of sources = 100 and $\tau = 100$ ms for all the flows. The packet-level simulations are done using a discrete event RCP simulator (for details, see [7]). The insights from the analysis suggest that when queue size feedback is included, one could expect the emergence of limit cycles with a larger amplitude as compared to the case where queue feedback is excluded. This insight bears out in simulations as well, where the case with queue feedback (see Figure 3(a)) has larger sized limit cycles as compared to the case without queue size feedback (see Figure 3(b)).

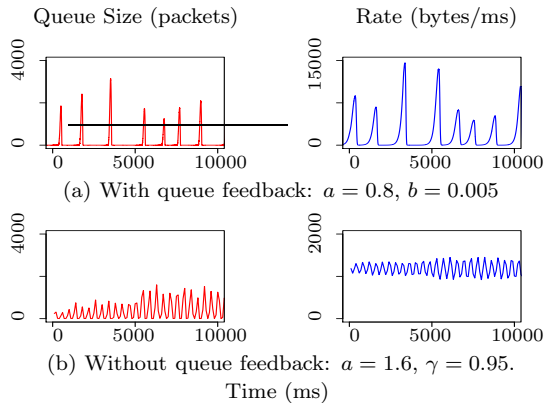


Figure 3: Simulation traces showing that the system which includes queue feedback exhibits limit cycles with amplitude that are much larger than that of RCP which uses only rate mismatch feedback.

6. CONTRIBUTIONS

The current design specifications of RCP recommend that it should estimate the fair rate of flows using feedback based on rate mismatch and queue size. Using a combination of analysis and packet-level simulations, we highlight that it would be beneficial to only have feedback based on rate mismatch. Given the non-linear nature of the model, we showed that the presence of two forms of feedback may, in fact, lead to a sub-critical Hopf bifurcation, which is undesirable in engineering applications. Whereas, in the absence of queue feedback, the Hopf bifurcation is always super-critical, and leads to the emergence of stable limit cycles. The results of our current study favors the design choice which uses feedback based only on rate mismatch in the design of RCP.

7. REFERENCES

- [1] A. Abuthahir, G. Raina and T. Voice. Do we need two forms of feedback in the Rate Control Protocol (RCP)? *arXiv:1906.06153 [cs]*, 2019.
- [2] H. Balakrishnan, N. Dukkipati, N. McKeown and C.J. Tomlin. Stability analysis of explicit congestion control protocols. *IEEE Communications Letters*, 11:823–825, 2007.
- [3] L. Baretto. XCP-Winf and RCP-Winf: improving explicit wireless congestion control. *Journal of Computer Networks and Communications*, 2015.
- [4] N. Dukkipati, N. McKeown and A.G. Fraser. RCP-AC: congestion control to make flows complete quickly in any environment. In *INFOCOM*, 2006.
- [5] B. Ermentrout. *Simulating, analyzing, and animating dynamical systems: a guide to XPPAUT for researchers and students*. SIAM Publications, 2002.
- [6] B.D. Hassard, N.D. Kazarinoff and Y.H. Wan. *Theory and applications of Hopf bifurcation*. CUP, 1981.
- [7] F. Kelly, G. Raina and T. Voice. Stability and fairness of explicit congestion control with small buffers. *ACM Computer Communication Review*, 38(3):51–62, 2008.
- [8] K. Lei, C. Hou, L. Li and K. Xu. A RCP-based congestion control protocol in named data networking. In *Conference on Cyber-Enabled Distributed Computing and Knowledge Discovery*, 2015.
- [9] M. Mahdian, S. Arianfar, J. Gibson and D. Oran. MIRCC: multipath-aware ICN rate-based congestion control. In *ACM Conference on ICN*, 2016.
- [10] G. Raina. Local bifurcation analysis of some dual congestion control algorithms. *IEEE Transactions on Automatic Control*, 50(8):1135–1146, 2005.
- [11] Y. Ren, J. Li, S. Shi, L. Li and G. Wang. An explicit congestion control algorithm for named data networking. In *INFOCOM*, 2016.
- [12] N.K. Sharma et al. Evaluating the power of flexible packet processing for network resource allocation. in *USENIX NSDI*, 2017.
- [13] S.H. Strogatz. *Nonlinear dynamics and chaos: with applications to physics, biology, chemistry, and engineering*. CRC Press, 2018.
- [14] T. Voice and G. Raina. Stability analysis of a max-min fair Rate Control Protocol (RCP) in a small buffer regime. *IEEE Transactions on Automatic Control*, 54(8):1908–1913, 2009.
- [15] L. Zhang et al. Named Data Networking (NDN) project. PARC, Technical Report NDN-0001, 2010.